# Angular dependence of the microwave-generation threshold in a nanoscale spin-torque oscillator

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It is shown that in a spin-torque microwave oscillator based on a magnetic nanocontact, the nature of the microwave spin wave mode generated at the threshold critically depends on the angle between the external bias magnetic field and the plane of the free layer. When the external bias field is rotating from normal to in-plane orientation, an abrupt transition from a propagating cylindrical wave with the frequency *higher* than the frequency of the linear ferromagnetic resonance (FMR) to a self-localized standing nonlinear spin wave "bullet" with the frequency *lower* than the FMR frequency takes place at a certain intermediate angle $\theta_{cr}$. This transition manifests itself as an abrupt jump (of the order of several gigahertz) in the generated microwave frequency. This mechanism of mode switching might explain abrupt jumps of the generated microwave frequency observed in recent experiments on spin-torque oscillators.

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## I. INTRODUCTION

It was predicted theoretically[1,2] and confirmed experimentally[3–8] that spin-polarized current passing through a thin magnetic layer ("free" layer of a magnetic layered structure) can excite microwave magnetization oscillations in this layer. Many experimentally observed features of the phenomenon were explained in a series of theoretical papers.[9–12] Although it is clear from experiments that the excitation of microwave oscillations has a threshold character (i.e., it is observed for sufficiently large currents $I > I_{th}$) and that the excitation frequency is close to the natural ferromagnetic resonance (FMR) frequency of the system, the exact nature of the dynamic spin wave modes excited at the threshold was not determined. This is especially true for the case of current-driven nanocontacts,[3–6] where magnetic layers are not bound in plane, and there are no reflective lateral boundaries that could create an effective magnetic resonator (like in the case of nanopillars with well-defined lateral sizes[7]).

The dependence of the microwave excitation in a magnetic nanopillar on the angle between the in-plane magnetization of the fixed (reference) layer and the in-plane direction of the external bias magnetic field in the free layer was studied numerically in Ref. 13. In that paper, the variation of the in-plane orientation of the bias field did not lead to any qualitative changes in the excited spin wave modes, but significantly influenced the transient temporal interval during which the system reached the steady-state microwave generation.

In contrast, in the case of the *nanocontact* geometry and *out-of-plane* orientation of the external bias magnetic field, the variations in the magnetization angle can lead to a qualitative change in the nature of the excited spin wave modes.

The theoretical analysis of spin wave excitation by spin-polarized current in a nanocontact geometry has been performed in two limiting cases: in the case of normally[9,14] and tangentially (in plane[15]) magnetized free magnetic layer. It turns out that the nature of the excited spin wave modes in these two cases is qualitatively different.

In the case of a normally magnetized film, both linear[9] and nonlinear[14] analyses predict that the spin wave mode excited by spin-polarized current is a propagating cylindrical spin wave with the wave vector $k_c \simeq 1.2/R_c$, where $R_c$ is the nanocontact radius. The coefficient of a nonlinear frequency shift $N$ in this geometry is positive $N > 0$ (see Ref. 11 for details), so that with the increase of the bias current $I$ (and the amplitude $a$ of the excited spin wave mode), the frequency of the excited spin wave mode also increases. Thus, the frequency $\omega(k)$ of this mode is always larger than the frequency $\omega_0$ of the FMR in the free magnetic layer:

$$\omega(k) = \omega_0 + Dk^2 + |N||a|^2, \tag{1}$$

where $k$ is the wave number of the excited mode, $\omega_0$ is the FMR frequency, and $D$ is the spin wave dispersion coefficient determined by the exchange interaction. The main contribution into the threshold current $I_{th}^{lin}$ of this propagating mode [see Ref. 9 and Eq. (2) in Ref. 15] is provided by the radiation losses proportional to the dispersion coefficient $D$. Therefore, the threshold current $I_{th}^{lin}$ in the case of normal magnetization does not vanish even in the case of an arbitrarily small damping.

In the opposite limiting case of an in-plane magnetized nanocontact, theoretical analysis performed in Ref. 15 demonstrated an absolutely different picture. In this case, the coefficient of the nonlinear frequency shift $N$ is negative, $N < 0$, and, therefore, has the sign that is opposite to the sign of the spin wave dispersion coefficient $D$ ($ND < 0$). Thus, apart from a linear propagating spin wave, the in-plane magnetized free magnetic layer can support a strongly localized nonpropagating spin wave mode of a solitonic type—spin wave "bullet."[15] Although it is well known that two-dimensional

 

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solitonic wave packets (bullets) in conservative systems are, generally, unstable (see, e.g., Refs. 16 and 17), the interplay between the spin-polarized current, creating negative damping, and natural positive magnetic damping of the free layer can stabilize a spin wave bullet of a certain amplitude $a_0$ and size $\ell$.[15,18] The frequency $\omega$ and the spatial size $\ell$ of the stable spin wave bullet excited by spin-polarized current are given by Eq. (6) from Ref. 15,

$$\omega = \omega_0 - |N||a_0|^2, \quad \ell = \frac{\sqrt{|D/N|}}{|a_0|}, \tag{2}$$

where $\omega_0$ is again the FMR frequency in the free layer.

Since the frequency of the spin wave bullet (2) is shifted by nonlinearity below the spectrum of propagating linear spin waves, the bullet mode has an evanescent nonpropagating character, and the radiation losses for the self-localized bullet vanish. As a result of this self-localization, the nonlinear spin wave bullet has a significantly lower excitation threshold $I_{\mathrm{th}}^{\mathrm{bul}}$ than a linear propagating mode [see Eq. (13) in Ref. 15 for details].

In this paper, we investigate theoretically the nature of spin wave modes excited by spin-polarized current in a magnetic nanocontact magnetized at an arbitrary out-of-plane angle. We assume that the angle $\theta_{\mathrm{int}}$ between the direction of static magnetization and the plane of the free magnetic layer in a nanocontact is varied continuously from 0 (corresponding to the case of in-plane magnetization) to 90°.

We demonstrate that the nonlinear nonpropagating bullet mode exists in a wide range of magnetization angles $0 \leqslant \theta_{\mathrm{int}} \leqslant \theta_{\mathrm{lin}}$, where $\theta_{\mathrm{lin}}$ is the "linear" magnetization angle at which the nonlinear frequency shift vanishes, $N=0$. We also show that below a certain critical angle $\theta_{\mathrm{cr}}$, the threshold bias current $I_{\mathrm{th}}$, necessary for the excitation of the bullet mode, is lower than the threshold of excitation of the linear propagating spin wave mode. In this range of magnetization angles $0 \leqslant \theta_{\mathrm{int}} \leqslant \theta_{\mathrm{cr}}$, the spin wave mode excited at the threshold will be the bullet mode, with the frequency lower than the FMR frequency, while for larger magnetization angles $\theta_{\mathrm{cr}} \leqslant \theta_{\mathrm{int}} \leqslant \pi/2$, the spin wave mode excited at the threshold is a linear propagating spin wave mode. The dependence of the excitation threshold $I_{\mathrm{th}}$ on the magnetization angle $\theta_{\mathrm{int}}$ is continuous, but demonstrates a kink at the transition between the two excited modes. In contrast, the angular dependence of the frequency $\omega_{\mathrm{th}}$ generated at the threshold has an abrupt jump of the order of several gigahertz at $\theta_{\mathrm{int}}=\theta_{\mathrm{cr}}$ in the transition region.

## II. ANALYTICAL MODEL OF CURRENT-INDUCED MICROWAVE GENERATION

To describe the spin wave mode generation in an obliquely magnetized magnetic nanocontact, we will use the standard Hamiltonian formalism,[17] successfully used before for the analytical analysis of spin-polarized phenomena.[10,11,15] Using this approach, one can derive an approximate equation for the dimensionless complex spin wave amplitude $a(t,\mathbf{r})$ of the variable magnetization normalized as

$$|a|^2 = (M_0 - M_z)/2M_0, \tag{3}$$

where $M_0$ is the length of the magnetization vector in the free layer and $M_z$ is the projection of this vector on the axis $z$ defining the direction of static magnetization in the free layer. This equation can be written in the form[15]

$$\frac{\partial a}{\partial t} = -i(\omega_0 - D\Delta + N|a|^2)a - \Gamma a + \sigma I f\left(\frac{r}{R_c}\right)(1-|a|^2)a, \tag{4}$$

where $\omega_0$ is the frequency of linear ferromagnetic resonance,

$$\omega_0 = \sqrt{\omega_H(\omega_H + \omega_M \cos^2 \theta_{\mathrm{int}})}, \tag{5}$$

$\omega_H \equiv \gamma H_{\mathrm{int}}$, $\omega_M \equiv 4\pi\gamma M_0$, $\gamma \approx 2.8$ MHz/Oe is the gyromagnetic ratio, $D \equiv \omega_M \lambda_{\mathrm{ex}}^2 (\partial\omega_0/\partial\omega_H)$ is the dispersion coefficient of spin waves due to inhomogeneous exchange interaction ($\lambda_{\mathrm{ex}} \approx 5$ nm is the exchange length), $\Delta$ is the two-dimensional Laplace operator in the film plane, $N$ is the nonlinear frequency coefficient given by

$$N \equiv -2M_0 \frac{\partial\omega_0}{\partial M_0} = \frac{\omega_H\omega_M}{\omega_0}\left(\frac{3\omega_H^2 \sin^2 \theta_{\mathrm{int}}}{\omega_0^2} - 1\right), \tag{6}$$

$\Gamma \equiv \alpha_{\mathrm{G}}\omega_0(\partial\omega_0/\partial\omega_H)$ is the linear Gilbert damping rate, $\sigma \equiv \epsilon g \mu_B / 2eM_0 dS$ ($\epsilon$ is the spin-polarization efficiency defined in Refs. 1 and 9, $g$ is the spectroscopic Lande factor, $\mu_B$ is the Bohr magneton, $e$ is the modulus of electron charge, $d$ is the thickness of the film, and $S=\pi R_c^2$ is the area of the nanocontact), $I$ is the charge current through the nanocontact, and $f(x)$ is the dimensionless function describing current distribution across the nanocontact. In this work, we will assume uniform current distribution, i.e., $f(x)=1$ for $x<1$ and $f(x)=0$ otherwise.

In Eqs. (4)–(6), $H_{\mathrm{int}}$ and $\theta_{\mathrm{int}}$ are the *internal* bias magnetic field magnitude and out-of-plane angle, respectively, connected with the *external* values $H_{\mathrm{ext}}$ and $\theta_{\mathrm{ext}}$ by the usual electrodynamic boundary conditions:

$$H_{\mathrm{ext}} \cos \theta_{\mathrm{ext}} = H_{\mathrm{int}} \cos \theta_{\mathrm{int}}, \tag{7a}$$

$$H_{\mathrm{ext}} \sin \theta_{\mathrm{ext}} = (H_{\mathrm{int}} + 4\pi M_0) \sin \theta_{\mathrm{int}}. \tag{7b}$$

We note that Eq. (4) for an *obliquely* magnetized nanocontact formally coincides with the equations derived for the case of *normally*[9] and *in-plane*[15] magnetized nanocontacts. The only difference is in the explicit expressions for the FMR frequency $\omega_0$, nonlinear frequency shift coefficient $N$, and the dispersion coefficient $D$. Therefore, we were able to easily generalize the results previously obtained in particular cases to a general case of an obliquely magnetized magnetic nanocontact.

## III. RESULTS AND DISCUSSION

We solved Eq. (4) numerically, assuming circularly symmetric profile and harmonic time dependence for the excited spin wave mode [i.e., using ansatz $a=a(r)e^{-i\omega t}$ in Eq. (4)], and calculated threshold currents corresponding to the excitation of a linear propagating spin wave mode (dashed line)

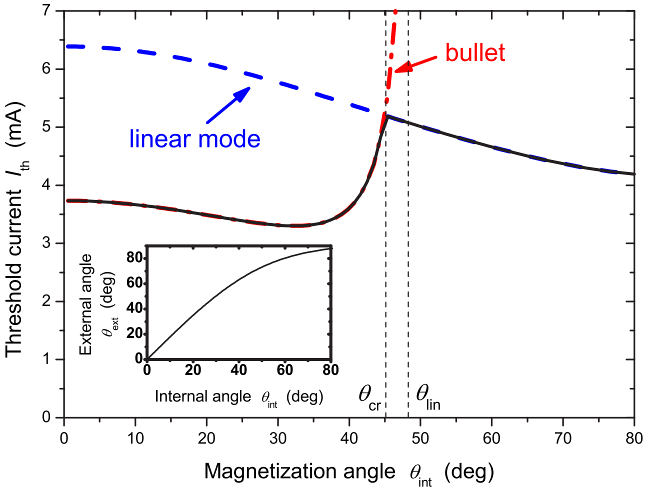


FIG. 1. (Color online) Main panel: Dependence of the threshold current $I_{th}$ on the out-of-plane angle of the internal bias magnetic field $\theta_{int}$. The dashed line shows the threshold current $I_{th}^{lin}$ of excitation of the linear mode, the dash-dotted line the threshold current $I_{th}^{bul}$ of excitation of the nonlinear bullet. Dashed vertical lines show the critical magnetization angle $\theta_{cr}$, separating regions of excitation of the bullet mode (for $\theta_{int} < \theta_{cr}$) and linear propagating mode (for $\theta_{int} > \theta_{cr}$), and "linear" magnetization angle $\theta_{lin}$, above which ($\theta_{int} > \theta_{lin}$) the self-localized mode does not exist. The parameters are $4\pi M_0 = 8$ kG, $H_{ext} = 10$ kOe, $\lambda_{ex} = 5$ nm, $L = 5$ nm, $\alpha_G = 0.02$, $\epsilon = 0.3$, and $R_c = 20$ nm. Inset: Dependence of the external out-of-plane magnetization angle $\theta_{ext}$ on the internal angle $\theta_{int}$. The parameters are $4\pi M_0 = 8$ kG and $H_{ext} = 10$ kOe.

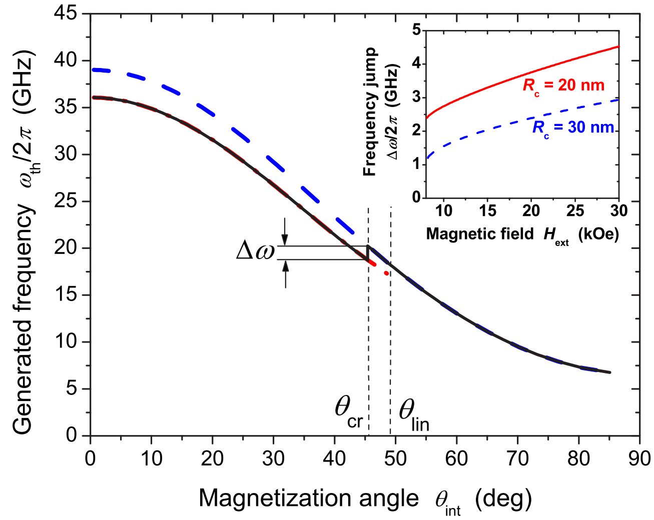


FIG. 2. (Color online) Main panel: Dependence of the frequency generated at the excitation threshold on the out-of-plane angle of the internal bias magnetic field $\theta_{int}$. The dashed line shows the frequency of the linear mode, the dash-dotted line the frequency of the nonlinear bullet. Vertical dashed lines show magnetization angles $\theta_{cr}$ and $\theta_{lin}$ (see caption of Fig. 1). The parameters are the same as in Fig. 1. Inset: Dependence of the frequency jump $\Delta\omega$ at $\theta_{int} = \theta_{cr}$ (see main panel) on the magnitude of the external bias magnetic field $H_{ext}$ for two different nanocontact radii: $R_c = 20$ nm (solid line) and $R_c = 30$ nm (dashed line). All other parameters are the same as in Fig. 1.

and standing self-localized nonlinear spin wave bullet (dash-dotted line) as functions of the internal out-of-plane magnetization angle $\theta_{int}$ at a fixed magnitude of the external bias magnetic field $H_{ext} = 10$ kOe. These two threshold curves for two qualitatively different spin wave modes are presented in Fig. 1. The inset in this figure shows the relation between the internal $\theta_{int}$ and external $\theta_{int}$ magnetization angles calculated from Eqs. (7a) and (7b) for $H_{ext} = 10$ kOe. The solid line in Fig. 1 gives the *minimum* threshold current at a given internal magnetization angle.

It is clear from Fig. 1 that at a certain intermediate internal magnetization angle $\theta_{cr}$ the nature of the excited spin wave mode abruptly changes, and the threshold curve $I_{th}(\theta_{int})$, although remaining continuous, demonstrates a characteristic kink at the transition point.

This abrupt transition from standing to propagating spin wave mode taking place with the increase of the magnetization angle $\theta_{int}$ manifests itself more dramatically in the calculated dependence of the spin wave frequency generated at the threshold on $\theta_{int}$ presented in Fig. 2. In this figure, the upper dashed line represents the frequency of the quasilinear propagating spin wave generated at the threshold [see Eq. (1) for $|a| \to 0$], while the lower dash-dotted line represents the threshold frequency of the nonlinear standing spin wave bullet [see the first equation of Eqs. (2)]. An abrupt jump in the frequency generated at the threshold (of the order of several gigahertz) takes place at the point of mode switching $\theta_{int} = \theta_{cr}$. In the inset in Fig. 2, it is shown that the magnitude of this frequency jump increases with the decrease of the nanocontact radius $R_c$ and, also, slowly increases with the increase of the external bias magnetic field.

The calculated dependence of the critical internal magnetization angle $\theta_{cr}(H_{ext})$ (at which the mode switching takes place) on the magnitude of the external bias magnetic field is shown in Fig. 3. It is clear from Fig. 3 that the angle $\theta_{cr}$ decreases with the increase of $H_{ext}$ and practically does not depend on the contact radius. It is also clear that the difference between this angle and the angle $\theta_{lin}$ (upper dash-dotted line in Fig. 3) at which the coefficient of nonlinear frequency shift $N$ goes through zero and changes its sign increases with the increase of the bias magnetic field magnitude.

So far, we considered the behavior of a magnetic nanocontact only at the threshold of microwave generation. In this case, changes in the internal magnetization angle $\theta_{int}$ can be caused only by the changes in the magnitude $H_{ext}$ or direction $\theta_{ext}$ of the external bias magnetic field.

In the above-threshold case, when the amplitude of the excited spin wave mode $a$ is substantial, there exists another possibility for the variation of the internal magnetization angle $\theta_{int}$. The increase of the mode amplitude (or the amplitude of microwave precession of magnetization) $a$ leads to the reduction of the static magnetization in the free layer $M_0 \Rightarrow M_z = (1-2|a|^2)M_0$, and, due to the boundary conditions [Eqs. (7a) and (7b)], to the changes in both the magnitude $H_{int}$ and direction $\theta_{int}$ of the internal bias magnetic field.

The decrease of the effective static magnetization $M_z$ in the free layer caused by the microwave precession will lead to the increase of the internal magnetization angle $\theta_{int}$. Therefore, if the "threshold" ($|a|=0$) operating point corre-

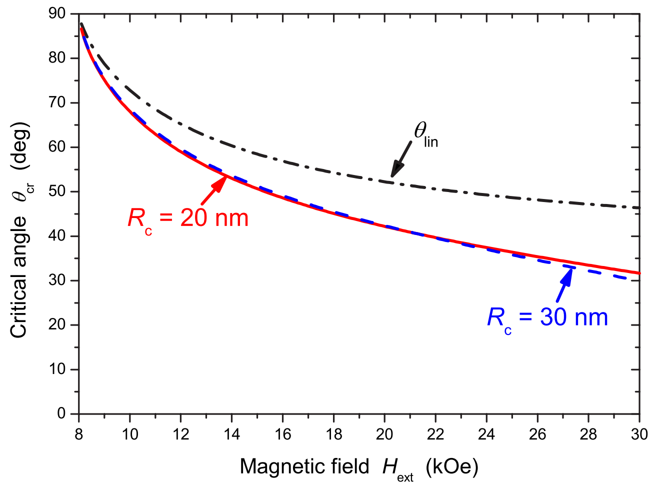


FIG. 3. (Color online) Dependence of the critical angle $\theta_{cr}$, at which transition from bullet to linear mode occurs, on the magnitude of the external magnetic field $H_{ext}$. Solid line, nanocontact radius $R_c = 20$ nm; dotted line, $R_c = 30$ nm. Dash-dotted line shows the angle $\theta_{lin}$, at which the nonlinear frequency shift vanishes, $N \to 0$ (this angle sets the upper limit of existence of the bullet mode). All other parameters are the same as in Fig. 1.

sponds to the region of excitation of a bullet spin wave mode, $\theta_{int} < \theta_{cr}$, the increase of the bias current and, consequently, the increase in the precession amplitude can cause transition to the region of excitation of a quasilinear propagating spin wave mode, $\theta_{int} > \theta_{cr}$.

As a result, with the increase of the amplitude of the bias current $I$, one could observe in experiments performed in the nanocontact geometry abrupt jumps in the generated microwave frequency, similar to the jump shown in Fig. 2.

## IV. CONCLUSION

In conclusion, we studied theoretically the nature of microwave spin wave modes that could be excited by spin-polarized direct current in obliquely magnetized magnetic nano-contacts. It was demonstrated that with the increase of the out-of-plane magnetization angle, an abrupt switching between two qualitatively different (propagating and standing) spin wave modes can take place. This switching should manifest itself as an abrupt change of the microwave frequency generated at the threshold of microwave excitation. We also believe that the proposed mechanism of spin wave mode switching might be responsible for the jumps in generated frequency observed in some of the recent experiments on microwave generation by spin-polarized current.[6,7]

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